

Deterministic Nonperiodic Flow (1963)

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This paper follows the simulation and physical construction of a dynamical system with magnetic interaction between its parts. The idea of large numbers of identical units giving rise to emergent patterns lead first to the experimental setup of two-dimensional dipolar spin systems. We observed certain characteristics of complexity ranging from simple neighbor-interaction in the micro-scale to dynamic pattern formation in the macro-scale. A kinetic art sculpture consisting of identical propeller-like units with magnets mounted onto the end of their arms was constructed and used in an interactive art installation. Participants were asked to interact by turning the units and therefore cause a spreading of movement and eventually pattern formation through attraction and repulsion.

I. INTRODUCTION

The work we did was based on our interest in creating a dynamical system with magnetic interaction between its parts. Our end goal was to take this system out of the computer by constructing a model to provide a simple and elegant physical embodiment of a dynamical system. The aim being to allow an individual to readily and intuitively observe some of the principles contained within the field of dynamical systems.

In a dynamical system the evolution rule states that there exists a fixed rule that describes what future states follow from the current state. Thus, an iterative process must be used to solve for the collection of future points known as a trajectory or orbit [1]. These trajectories, and dynamical systems in general, are highly dependent upon their initial conditions. Small initial variations may turn out to effect large variations in the long-term behavior of the system. This effect was evident in our models. In both the computer simulations and the physical kinetic sculpture various initial conditions produced widely varying patterns.

II. MOTIVATION

In discussing the motivation behind our work it makes sense to first touch on the areas of interest that provided our starting point – as well as mention the artists dealing with these fields who influenced our work. I will then wrap up by coming back to the idea and the goal of our work.

The three areas we wished to explore with our work were patterns, kinetic art and magnetism.

A. Patterns

To talk of a particular artist that uses patterns is an impossible task. It is obvious that in all fields artists

have been fascinated by the beauty of patterns, from the geometric designs of some of our earliest known artworks to the work of Sol LeWitt and beyond. One common feature of patterns is self-similarity. This self-similarity can range from rough, as in many natural phenomena, to perfect, as is the case in cellular automata, for example, where you have identical units following identical rules. We were inspired by this idea of identical units giving rise to emergent patterns in our own work. Like Conway's *Game of Life* (1970), our computer simulation consisted of lattices of identical individual units, in our case dipoles, which then gave rise to emergent patterns. As a side note, it is worth mentioning that in our case, unlike classical cellular automata, we were dealing with continuous automata without discrete rules and states. In any case, the key idea we wished to explore was, as discussed by Ball, the fact that complex patterns do not necessarily require painstaking human labour. And that, in fact, many beautiful, complex, and fascinating patterns are the result of incredibly simple processes and basic physical laws—one common example being the perfect hexagonal honeycomb pattern found in the honeybee nest [2].

B. Kinetic Sculpture

In order to transfer our idea of autonomous units and pattern generation from computer to the 'real world' it was necessary to build a dynamic kinetic structure. The general concept of motion as art is not particularly new. From the free flight of a bird, to highly structured tribal dance, movement has long captured the human imagination. In terms of a recognized field of art, however, it was not really until the emergence of kinetic art in the past century that movement and 'museum' began to come together. There are many types of kinetic art, but our structure follows most closely the work begun by the inventor of the mobile, Alexander Calder.

In terms of present day artistic influences we found the art of both Tim Prentice and Theo Jansen quite captivating and relevant. In pieces such as *Carpet* and *Maquette* Prentice constructs his kinetic sculptures out of hundreds of individual identical components, which, together, give rise to fascinating movement that, in his words, “make the air visible” [3]. Jansen, similarly, makes wind-powered ‘beach animals’, which, though less modular than Prentice’s work, have particular relevance in that a large part of the artists goal was to transfer his visions from the computer to the physical [4]. Given these similarities it is clear that the most obvious structural difference for our own kinetic sculpture is the mode of propulsion, where, rather than air movement, we employ the force of magnetism.

C. Magnetism

Magnetism in kinetic sculpture is not something new - from Greek kinetic artist Takis’ *Télésculpture* (1959), which introduced the idea of magnetic levitation [5], to Pol Bury in his *2000 Billes sur un Plateau* (1971), to present day work like David Durlach’s dancing iron dust, or Sachiko Kodama’s *Protrude, Flow* (2001). Our own structure does not share any obvious close commonalities with these works, but we were definitely inspired by the sense of wonder that the effects of magnetism can induce, when used in novel and creative ways.

D. Synthesis

Our goal was to take all of these influences and interests: patterns, kinetic sculpture, and magnetism, as a means of creating a model that could surprise us. More specifically we wished to examine the idea of many identical parts producing an unpredictable result, and the concept of small differences in initial conditions manifesting themselves as large variations – in this case the movement of the various arms of our model.

A huge number and variety of systems possess a sensitive dependence on initial conditions. From famously complex systems such as the weather, to something as simple as a pachinko game, we are surrounded by this phenomena and are intuitively aware of its two key factors: slight differences in initial conditions can lead to widely divergent outcomes, and, the farther forward we attempt to look the more uncertain our predictions become.

The audience can experience this same phenomenon on an intuitive level with the small-scale dynamical system we have constructed. For example, when one of the arms is turned we are not surprised that this initial motion in turn affects other arms. However, the

actual path of the movement across the grid is highly unpredictable. Sometimes there is almost no motion, and at others we can see ‘lines’ of motion extending to the opposite edge.

At the same time as it is unpredictable, we also recognize that it is not irrational. Additionally, after a bit of time we can come to see that, in time, recognizable patterns do in fact appear and we can come to a limited understanding, at least in terms of the immediately adjacent arms, as to what movement will result from the turn of a particular arm.

In many ways this can be seen as a metaphor for life itself. Every human being is a complex system, both physically and mentally and the world we find ourselves immersed in is composed of many overlapping complex systems: nature, society, the economy and so on. In each of these systems we are acutely aware of both how great an impact a small change in condition may have in the future, as well as the increasing difficulty of prediction the farther into the future we attempt to look.

III. TWO-DIMENSIONAL DIPOLAR SYSTEM

Two-dimensional lattices of magnetic dipoles in different shapes, sizes and dimensions give a good experimental setup for studying the characteristics and abilities of magnetic force. As in classical dipolar spin systems the dipole moments are confined to rotate in the plane of the lattice. Through simple dipole-dipole interaction emerge different complex patterns, when the spinning dipoles find their equilibrium configuration. We studied those phenomena using a computer simulation and a physical model.

A. Simulation

The simulation shows a two-dimensional xy system of freely rotating point dipole sources on fixed locations in square and hexagonal lattices. The dipoles are constrained to rotate in the plane of the lattice [6]. Using the formulas for calculating the magnetic field (1) of every neighbor (2) and the resulting torque (3) on the magnetic dipole, all the magnetic particles in the lattice are updated in parallel. The dipole-dipole interaction shows chain reactions and a dynamic pattern formation over time.

The equation (1) for the magnetic field \mathbf{B} at a location given by \mathbf{r} includes the permeability of free space μ_0 and the magnetic dipole \mathbf{m} positioned at the origin. Using this equation we can visualize the magnetic field of a single vector as in Figure 1.

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} (3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}) \quad (1)$$

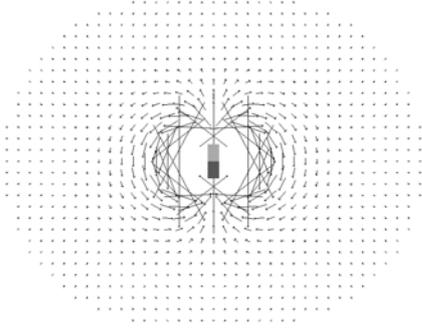


FIG. 1: A simulated magnetic field. The vector lengths visualize the strength of the magnetic forces.

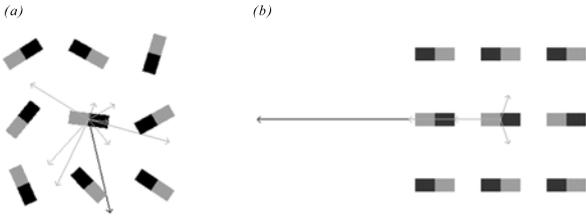


FIG. 2: Magnetic dipole force vectors.

The magnetic force on one magnet is the sum of the forces of the magnetic fields of all its neighbors on it.

$$\mathbf{B}_t(\mathbf{r}) = \sum_{\text{neighbors}} \mathbf{B}(\mathbf{r}) \quad (2)$$

In Figure 2 we can see a 3×3 magnetic dipole grid. In (a), which depicts a random rotational start configuration, the grey vectors show the force of the magnetic fields of the center magnets eight neighbors. The black arrow is the sum of those vectors. We see in (b) that if the magnetic dipoles line up, then so too do the force vectors. A minimal potential energy is reached when the vector-sum of the magnetic forces of the neighbors points in the same direction as the dipole moment.

The torque $\boldsymbol{\tau}$ on a spinning magnetic dipole is the cross product of the magnetic field \mathbf{B} and the magnetic dipole \mathbf{m} .

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \quad (3)$$

The torque $\boldsymbol{\tau}$ can also be defined as the time derivative of the angular momentum \mathbf{L} .

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} \quad (4)$$

The angular momentum L for a body that is rotating about a fixed symmetry axis is expressed as the

product of the moment of inertia I of the body and its angular velocity ω .

$$L = I \cdot \omega \quad (5)$$

Considering the fact that our model only allows the rotation around one axis, only an angle-value instead of a vector value is needed. Therefore we simplify the torque formular and are able to combine it easily with the angular momentum equation:

$$\tau = \frac{dL}{dt} = I \cdot \dot{\omega} \quad (6)$$

This allows us to find the angular acceleration $\dot{\omega}$:

$$\dot{\omega} = \frac{\tau}{I} \quad (7)$$

The ordering of the magnetic dipoles from an initial random rotation state depends on the dipolar strength, the assumed moment of inertia, the density of the two-dimensional lattice and an added friction value. The magnetic dipoles take a long time to slow down if their dipolar strength is too high or the friction value is too low. If this is not the case, the dipoles tend to slow down into smaller domains of pattern configurations. When those domains collide they either merge into bigger domains, if they share the same pattern, or one of the domains extinguishes the other. After a while the lattice finds his equilibrium configuration.

Traditionally you would assume that systems in an equilibrium condition would either be completely homogeneous or at least consist of large domains with homogenous structures. But it occurs frequently that equilibrium states show a mesoscale heterogeneity (modulated phases, patterns, etc.) of either regular (e.g., stripes, stripe domains) or ‘chaotic’ states [7].

Those equilibrium states observed in our simulation consist of different pattern elements. As stated by Stambaugh the anisotropic (directionally dependent) nature of dipolar interactions induces head-to-tail alignment, which results in the formation of linear, ring or branched structure [8].

Based on the structure, geometry and dimension of the lattice, different patterns emerge. The magnetic dipoles self-organize into their favored equilibrium configuration. Square lattices produce micro-vortices if their dimensions are small or concentric ring formations mixed with micro-vortices, if they are larger. Hexagonal lattices produce macro-vortices if the range of interaction is not restricted to the nearest neighbors.

Simulations of square lattices of up to 15×15 magnetic dipoles ultimately end up in a micro-vortex pattern after starting from a random spin configuration.

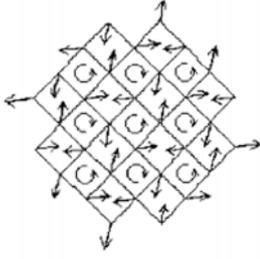


FIG. 3: A schematic illustration of the ‘square-packed micro-vortex’ pattern [8].

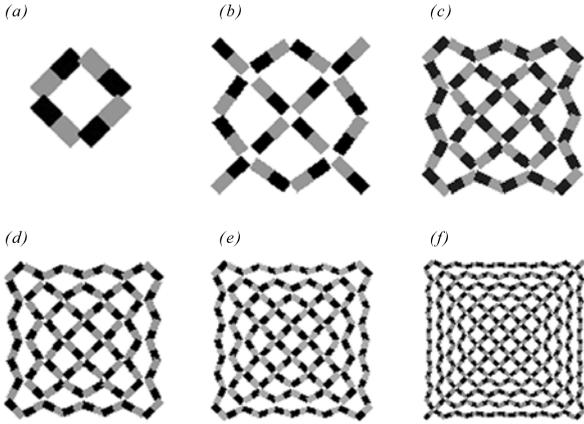


FIG. 4: Equilibrium states of square lattices with the size of (a) 2×2 , (b) 4×4 , (c) 6×6 , (d) 8×8 , (e) 10×10 , and (f) 15×15 magnetic dipoles.

A micro-vortex pattern consists of 4 neighboring magnetic dipoles that line up in a circle. Due to dipole orientations micro-vortex circles next to each other are oriented in opposite directions (see Figure 3). Those configurations have a very low local potential energy and are therefore very stable.

Square lattices with sizes larger than 15×15 already start to form macro-vortices and ring structures in their equilibrium states. The results show combinations of micro-vortex patterns and concentric ring formations (see Figure 4). Several vortex centers can appear and disappear during the simulations process before the lattice settles down in a stable configuration. Vortices are energetically favorable because in such configurations each spin is aligned along the dipolar field [9].

The vortices that appear in square lattices have centers that consist of four dipoles with their magnetic moments pointing into the same direction as you can see in Figure 6. The magnetic dipoles form rings around those centers. Each ring-chain has the opposite dipolar orientation from that of the two neighboring chains. A confirmation for this behavior can be found in Stambaugh’s experiment with small hard

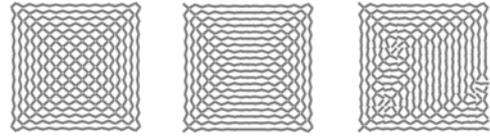


FIG. 5: Equilibrium states of square 20×20 dipole lattices starting from random spin configuration, with identical initial parameters.

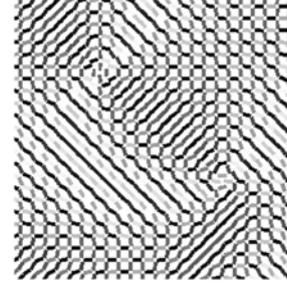


FIG. 6: A segment of a square lattice with color representing rotation angle.

spheres containing magnetic cores (see Figure 7).

Larger lattices, approaching infinite sizes (simulated by periodic boundary conditions), contain more vortex centers. Every vortex center becomes a crossing for 4 antiferromagnetic domains where the magnets are aligned with the horizontal or the vertical axis of the lattice. Those perpendicular aligned domains get connected on the edges with micro-vortex patterns. Figure 8 shows the aligned domains in white and the micro-vortex domains in grey color. The edges between those neighboring domains show smooth curved boundaries.

The patterns that emerge from the ordering process of the dipoles are very dependent on the magnetic dipole strength (see Figure 9). Also the dynamic characteristics of the formation process depend on the dipole strength. Considering a setup where the magnetic field vector of each dipole is only calculated from the sum of the forces of its 8 nearest neighbors, too weak and too strong dipole moments make the pattern formation more difficult. Weak dipole moments cause the dipoles to turn very slowly and to take a long time until they find a stable equilibrium. If the dipole moment is too strong the dipoles spin too fast and the never-ending feedback-loop makes it hard for the simulation to slow down in an equilibrium state.

Widening the range of interactivity changes the equilibrium states of the lattice. The very ordered domains of lined up dipoles become more complex the more neighbor forces are taken into account. Testing different ranges of interactivity on a finite square lattice shows that the simplicity of the nearest-neighbor-interaction-only patterns gets destroyed (see

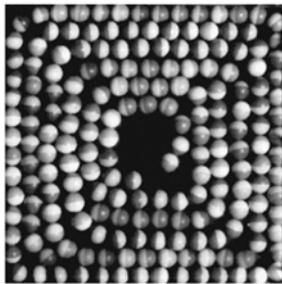


FIG. 7: Magnetic particles in hardsphere cubes line up in concentric rings after being excited by shaking [8].



FIG. 8: Equilibrium state of square 200×200 dipole lattice with periodic boundary conditions and random spin configuration start.

Figure 10).

Running the simulation on a hexagonal lattice produces aligned domains along the 3 hexagonal axes (0° , 120° , 240°), if only the 6 nearest neighbors of each magnetic dipole are taken into the calculation. As in the simulations of square lattices every line-chain has the opposite orientation of its two neighboring chains. The bigger the interactivity range is, the more neighbors are included and the more the emerging equilibrium patterns changes. When the radius of interactivity includes the 12 nearest neighbors (see Figure 11) line-domains still appear along the 3 hexagonal axes, but they are separated by bigger areas of clusters of macro-vortex centers.

It has been proven that the zero-temperature ground state of a finite two-dimensional hexagonal lattice of dipoles is a macrovortex, while the zero-temperature ground state of a finite square-packed lattice of dipoles is a microvortex [8]. Compared to the concentric rings / macro-vortex combination that showed up in larger square lattices, the macro vortices in the hexagonal lattices don't have a large dimension. The vortex centers are surrounded by concentric hexagonal rings around them. In contrary to the concentric rings in the square lattice, the dipole

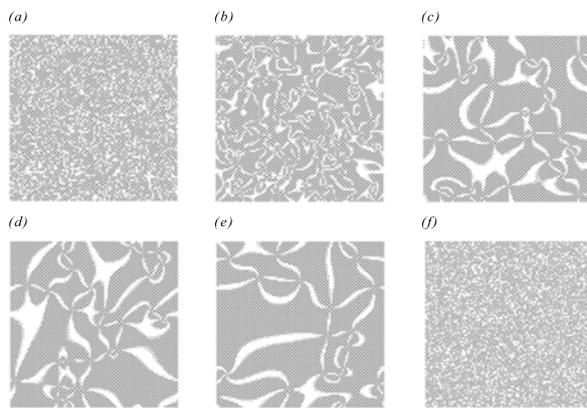


FIG. 9: Six square 100×100 dipole lattices with different dipole moment values, rising von (a) to (f).

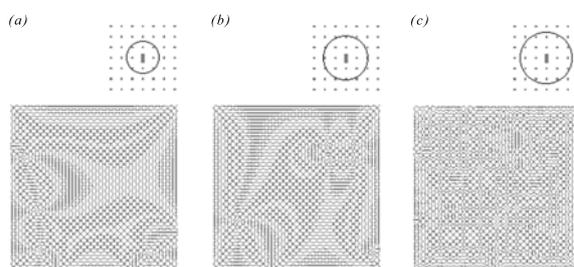


FIG. 10: Square 50×50 lattice calculated using (a) 8, (b) 12, and (c) 20 neighbors.

orientation of the hexagonal rings is the same. Depending on the range of interactivity there are different amounts of hexagonal rings around the centers. Lattices that include the dipole-dipole interaction of up to their 440 nearest neighbors (see Figure 12a) show up to 8 concentric hexagons around their centers. Those hexagonal forms of concentric rings are located next to each other and are also flowing into each other. The brightness of the coloring indicates the rotational angle of the dipole moments. Visualizing the rotational angle of the magnetic dipoles just in two colors (above and below 180°) shows a branching line-pattern (see Figure 12b).

B. Physical Model

A simple two-dimensional model of freely rotating magnets on fixed positions was setup. The experimental model setup consists of a board with holes along a 2.5×2.5 cm grid. Cylindrical permanent magnets were put into little holders on needles and were able to rotate freely in their holes (see Figure 13). We used magnets made of NdFeB N35, with a Ni-Cu-Ni surface, a maximal magnetic energy-density of $263\text{--}287$ kJ/m³, the ability to lift up to 3.5 kg and a size of

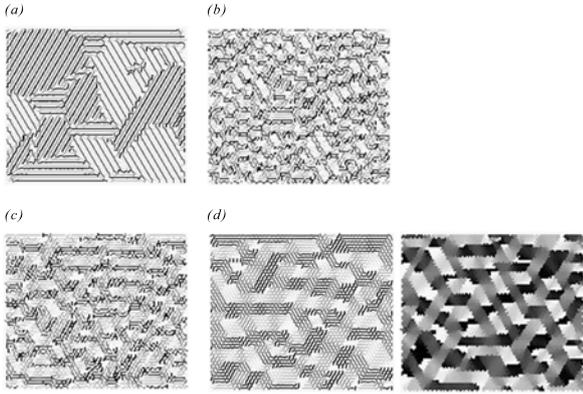


FIG. 11: Hexagonal 50×50 lattice calculated using (a) 6, (b) 12, (c) ≈ 50 , and (d) ≈ 100 neighbors, with brightness indicating dipole rotational value.

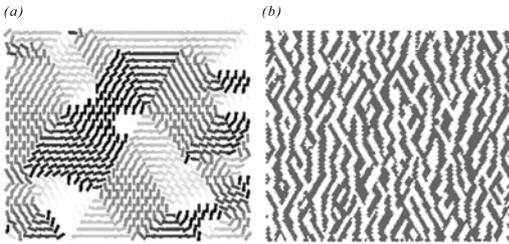


FIG. 12: (a) Detail from a section of a hexagonal 100×100 lattice calculated using ≈ 440 nearest neighbors. (b) Hexagonal 100×100 lattice calculated using ≈ 50 nearest-neighbors.

1 cm \times 0.5 cm (The magnets were purchased online at powermagnetshop.de).

When the magnets are put in their holders, the holders immediately rotate according to the magnetic fields of their neighbors. The resulting configurations show the same patterns as a similar system of point dipoles simulated on the computer. The rotating magnets line up in micro-vortex circles (See Figure 14). The model also shows the alternating orientation of neighboring micro-vortex circles.

IV. ROTATING UNITS

Taking the results of the two-dimensional dipolar system in the direction of kinetic art, we decided to construct a sculpture consisting of several propeller-like units that are able to rotate freely when being pushed by an external force. Those units are positioned on a fixed grid. Attaching small cylindrical magnets to the ends of the three-armed units gives the possibility of interaction between neighboring units. To investigate possible scenarios of different grid sizes, grid shapes, magnets strengths, amounts of

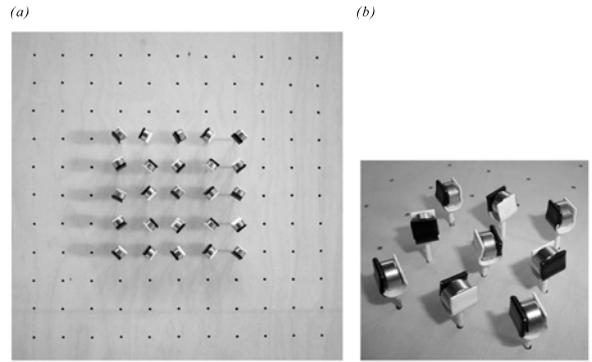


FIG. 13: (a) Overhead picture of board with rotating magnets. (b) Close-up of magnet and holder construction.

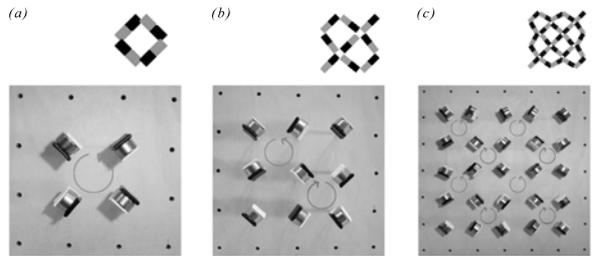


FIG. 14: Square lattices of (a) 2×2 , (b) 3×3 , and (c) 5×5 exhibiting micro-vortex patterns.

arms, length of arms or orientation of the attached magnets we first simulated the setup on the computer, before we constructed the actual sculpture.

A chain reaction of movement can be activated by spinning one rotating unit.

A. Simulation

To calculate the influence of the neighboring magnetic fields on the rotation of the propeller-units, first the magnetic field acting upon the magnet at the end of each arm must be calculated. We use the magnetic field formula (1) for every neighboring propeller-unit and every magnet located on the neighbors three arms. Then we transform this sum into a torque force acting upon the center of the rotating unit. This can be visualized in Figure 15 where the grey vectors represent dipole moments, and the dark vectors represent the magnetic field acting upon the dipole.

As the magnetic fields in this setup are not stable, but moving in space, another equation must be added. The rotation of the propeller-units constantly changes the existing magnetic fields by changing the position of the dipole moments. This causes another force that adds torque around the middle axis. The equations for calculating the force produced by the constantly

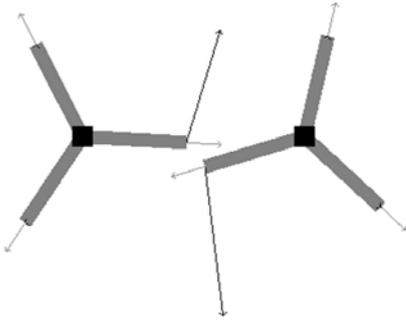


FIG. 15: Three-arm rotating units, with magnetic dipoles attached to the end of each arm.

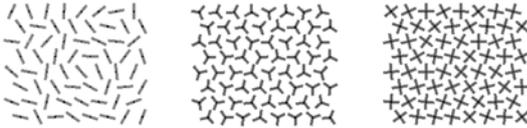


FIG. 16: 2, 3 and 4-armed rotating unit simulations.

changing magnetic field of one dipole \mathbf{m}_2 acting upon another \mathbf{m}_1 :

$$\mathbf{F}_{\vec{r} \rightarrow \mathbf{m}_1} = \vec{\nabla}(\mathbf{m}_1 \cdot \mathbf{B}_{\mathbf{m}_2 \rightarrow \mathbf{m}_1}) \quad (8)$$

Using the simulation to test different start parameter, several conclusions concerning size, shape and positioning of the system have been made. Varying the amount of arms that are attached to the rotating unit and studying the dynamics of each setup showed that a three-armed version gives the best dynamic action and reaction distribution (see Figure 16). Testing different grid forms showed that a hexagonal / triangular lattice allows the best distribution of movement. Every rotating unit has the ability to interact with its 6 nearest neighbors, whereas a square grid gives a setup with only 4 nearest neighbor units.

Alternating the distance between the rotating units had an immense effect on the dynamic behavior of the system. The magnetic field force of magnets decreases as a factor divided by the distance to the power of three and therefore already small adjustments show a big loss of force.

Figure 17 shows the results of a different orientation of the magnetic dipoles in the system. Here the dipole moments point perpendicular to the direction of the arms they are attached to. Therefore the side-attraction of the magnets makes them attract each other strongly and the arms align to form stable patterns like hexagonal rings and labyrinth-paths.

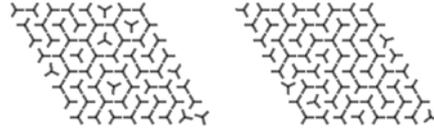


FIG. 17: Two hexagonal lattices with perpendicular dipole orientations.

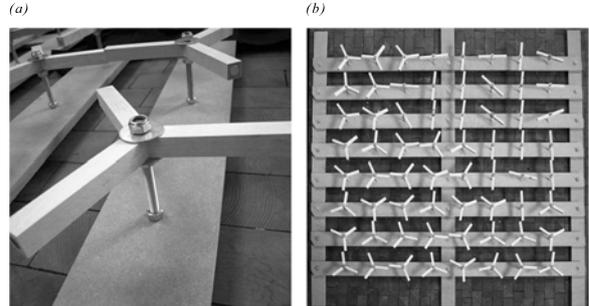


FIG. 18: (a) Close-up of an individual rotating unit. (b) The prototype in its square configuration.

B. Physical Model

The physical equivalent of the simulation was constructed out of wooden and metal parts. The basic structure of the model is a grid made of wooden boards with the dimensions of 2x2 meters. The shape of the square lattice can easily be changed by shifting the boards into another configuration, like the parallelogram at 30° which we chose for our installation. Onto each of the 8 boards 8 rotating units were mounted. The 64 units consist of a round metal axis and custom made wooden three-armed propellers with a hole in their center. The propeller-pieces are placed at a height of about 10 centimeters and are able to freely rotate around the axis when pushed. At the end of each wooden arm a cylindrical magnet is placed into a drilled hole. The distance between the rotating units provides them from touching each other, but is short enough for the magnets to attract and repel each other.

V. KINETIC/INTERACTIVE SCULPTURE

Our physical model was used in the parallelogram setup for the interactive installation, so the rotating units were aligned along a hexagonal grid and each of them could easily interact with its 6 nearest neighbors.

Through continuous rotation of one unit, the 6 units around it are pushed to the outside and end up forming a hexagonal pattern around the center (see Figure 19). Through the side-attraction of the magnets the arms of the neighbor units almost line up and are

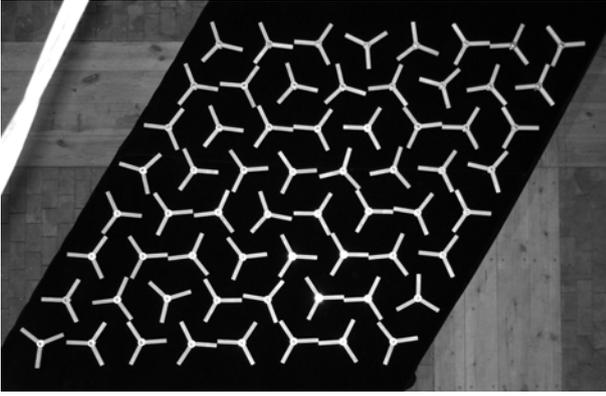


FIG. 19: Overhead view of the sculpture: hexagonal rings and labyrinth paths are clearly visible.

in a quite stable, but yet easily destroyable configuration. A slight torque might push the arms from sideways-attraction to pole-pole repulsion and therefore cause a forceful motion.

To emphasize the present forces during a movement in the sculpture we chose to amplify the vibrations that were caused by the spinning of the rotating units. Therefore we attached 2 piezo-microphones to the bottom side of each of the 8 boards the units were mounted on. The vibration signals were fed into self-powered sound speakers. This made it possible to amplify every little jittering of the sculpture and formed an immersive soundscape.

To play further with the propagation of movement in the sculpture the whole setup was captured with a video camera from above and a processed video image was projected onto the wall next to the sculpture. The projection on the wall showed a black screen when the sculpture was unmoved. But when participants turned one of the units they could follow the spreading of movement visualized by the appearance of white rotating units on the videoimage. The effect was produced by applying a motion detection filter to the original video image taken from above. The flickering and illuminating of the figures on the screen added a subtle and aesthetical layer to the interactive installation. The videoprojection caught the interest of participants and brought them closer to inspect the source of the visuals.

VI. CONCLUSIONS

Our project was a success in that it accomplished what we set out to do. We simulated and then constructed a unique dynamical system. It demonstrated a number of phenomena we wished to explore, such as pattern formation, magnetic forces and bifurcations. The computer simulation allowed us to test different configurations and to flexibly alter parameters—which

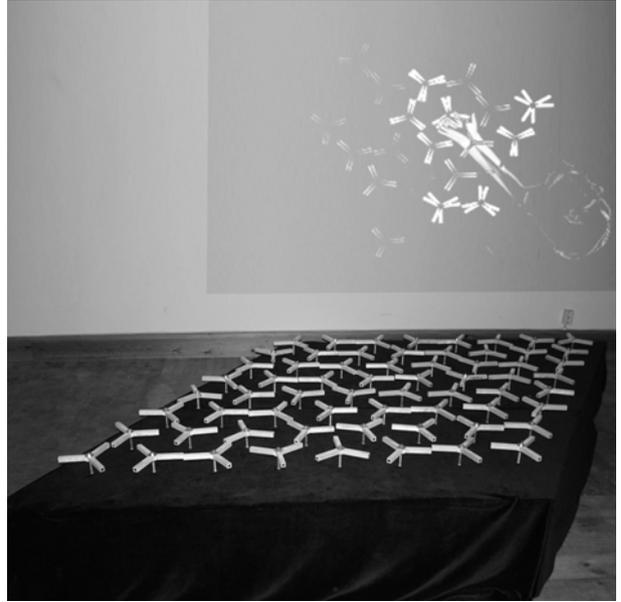


FIG. 20: The installation set-up, with wall projection.

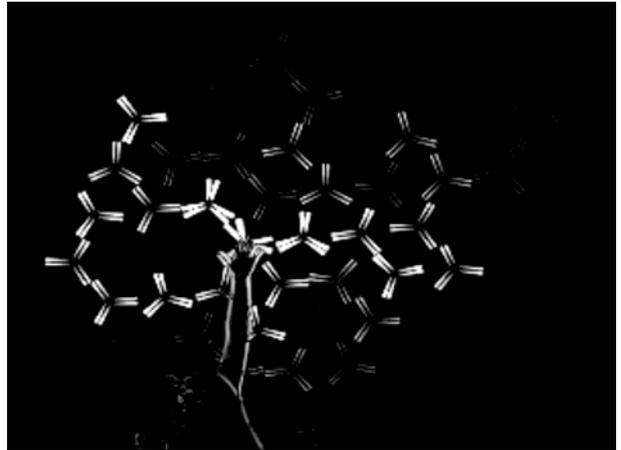


FIG. 21: The projected motion-detection image.

in turn helped us to gain a better overall understanding of how a dynamic magnetic lattice may act. The physical model was effective in that it gave the participant the means of a direct and intuitive interaction with a simple dynamical system, combined with an interesting aesthetic experience.

Perhaps one of the most interesting—although of course not totally surprising—things we came to understand over the course of the project was the challenge of trying to translate from real world to computer and vice versa. Both posed problems. On the computer simulation side, it is, not surprisingly, difficult to model physical dynamical systems in a way that accurately relates to the real world. Often parameters are set and changed until they ‘look right’, as it is either impractical or impossible to find ex-

act values. Then, taking the model from computer simulation to tangible reality is a dubious undertaking. In our case, ignoring any inaccuracy in the simulation, despite our best efforts the transfer process was compromised due to the imprecision of component parts and human error. In building this type of custom model the parts you are looking for do not already ‘exist’, so you are forced to either have the parts custom made, or, as in our case, choose the less expensive route of jerry-rigging an imperfect, albeit working, model out of readily available components.

This was not necessarily a negative, as we were able to incorporate some of the imperfections in an organic manner. For instance, the fact that we did not have precision bearings for the arms to rotate on and instead used a combination of washers, bolts and nuts meant that the arms did not always rotate on a perfectly horizontal plane—making a fair amount of noise in the process. We decided to use this noise, and to amplify it, and the effect, in our opinion, was that it gave the whole system more of a natural feel—in fact adding to the experience.

There are numerous ways in which we can envision extending this project. With the motion tracking we have now it is a fairly simple matter to represent the motion data in other, visually interesting ways. Map-

ping the speeds to a color gradient and projecting it back directly on the rotating units was one idea we have considered. Another idea along the same lines would be to have a pattern recognition program that would look for stable configurations (attractors). Another concept would be to have another process put in action when a certain threshold of motion is reached after the turning of one of the arms. Finally, we would also like to work on this same concept on a larger scale. In simulation larger grids—especially once we got over 50×50 —were much more visually interesting than those that were considerably smaller, with their mini and macro-vortexes. We would like to work on this scale, and at the same time test out other configurations for the individual units, in order to create increasingly complex dynamical system representations.

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